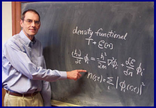


From damping of single particle resonances to exotic nuclei breakup: following George in the 'continuum'

Angela Bonaccorso

INFN, Sez. di Pisa

Symposium on the occasion of George Bertsch's 70^{th} birthday

Plan of the Presentation

Who is George?

George is a leading nuclear physicists able to work and make the links between structure and reaction aspects of Nuclear Physics.

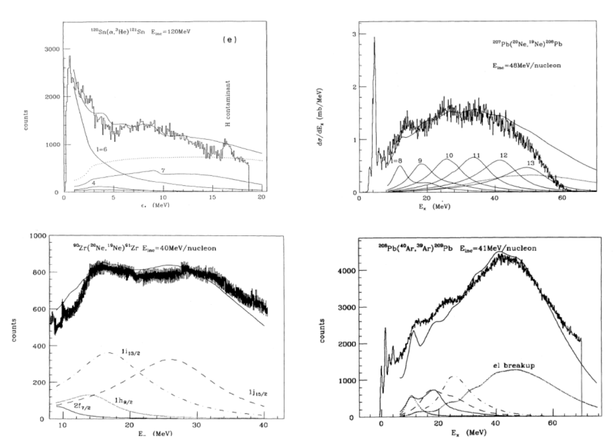
My contribution today has to do with reaction studies aiming at clarifying limits of the single particle concepts.

Key words will be:

Shell model, single particle, damping, optical potential, bound states vs continuum, transfer and breakup.

1989: begining of the story

During a sabatical at MSU in 1988, David Brink has a chat with Gary Crawley and Sydney Galès who show him spectra of this type. [AB, PRC51(1995) 822]



David writes to me asking whether I could do some calculations. September 1989: George invites me for two weeks. We meet for the first time. During that stay I understand that the n-target S-matrix has to be calculated with a time dependent optical potential, which will allow also to understand the WIDTHS of high energy single particle in the continuum.

Problems

Understand the shape, width and physical 3-body background.

Inspired by "Damping of Nuclear Excitations", Rev. Mod. Phys. 55, (1983) 287 G. F. Bertsch, P. F. Bortignon, R. A. Broglia

distributions to Gaussian functions,[1] although there is no theoretical justification for such a form. The suppression of the strength function wings in a Gaussian fit means that there will be a tendency to underestimate the total strength associated with the peak. It is important to use the best possible functional forms when attempting to extract a total strength of a peak that is partly obscured by a poorly understood background.

$$\Gamma = \Gamma^0 + \Gamma^{\downarrow}$$

$$\Gamma^{\downarrow} = -2 < W > = -2 \int \rho(r) W(r) d^3 r$$

$$\rho(r) = |\psi(r)|^2 \approx 0.08 fm^{-3}/A$$

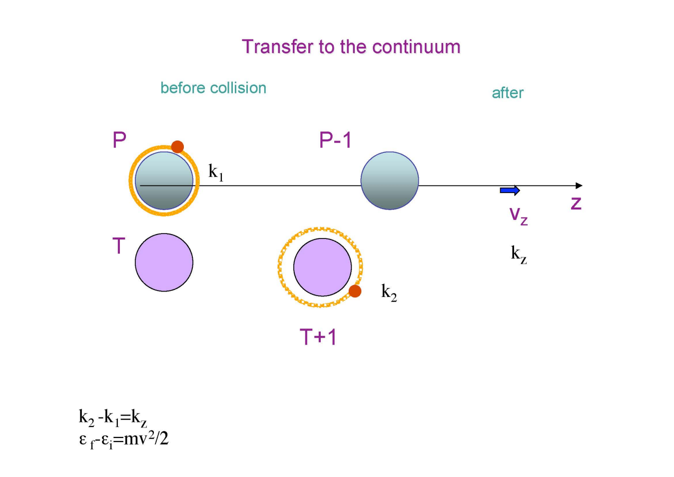
Brown-Rho parametrization

$$[r^2]_W^{BR} = \frac{4\pi}{A} \int r^2 dr W(r, \varepsilon_f) = -b_2 \frac{(\varepsilon_f - E_f)^2}{(\varepsilon_f - E_f)^2 + r_2^2}$$

TABLE II. (a) Spreading width as a function of the continuum energy in ^{208}Pb. (b) Spreading width as a function of the continuum energy in ^{91}Zr. All units are in MeV.

(a)								(b)	
ε_f	$\Gamma^{\downarrow}$	ε_f	$\Gamma^{\downarrow}$	ε_f	$\Gamma^{\downarrow}$	ε_f	$\Gamma^{\downarrow}$	ε_f	$\Gamma^{\downarrow}$
1	3.1	11	7.9	21	9.7	31	10.4	1	9.7
2	3.7	12	8.2	22	9.8	32	10.5	5	10.14
3	4.4	13	8.5	23	9.9	33	10.5	10	10.67
4	5.0	14	8.7	24	10.0	34	10.6	15	11.21
5	5.5	15	8.9	25	10.1	35	10.6	20	11.74
6	6.1	16	9.1	26	10.2	36	10.6	25	12.28
7	6.5	17	9.2	27	10.2	37	10.7	30	12.81
8	6.9	18	9.3	28	10.3	38	10.7	35	13.35
9	7.3	19	9.5	29	10.3	39	10.7	40	13.88
10	7.6	20	9.6	30	10.4	40	10.7		

Transfer to the continuum reaction (inclusive)



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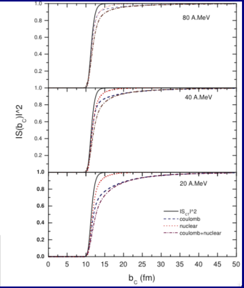
A consistent formalism for various breakup reactions

The core-target movement is treated in a semiclassic way, but neutron-target and/or neutron-core in a full QM treatment.

AB and DM Brink, PRC38, 1776 (1988), PRC43, 299 (1991), PRC44, 1559 (1991).

$$\frac{d\sigma}{d\varepsilon_f} = C^2 S \int_0^\infty d\mathbf{b_c} \frac{dP_{-n}(b_c)}{d\varepsilon_f} P_{ct}(b_c),$$



Use of the simple parametrization
$P_{ct}(b_c) = |S_{ct}|^2 = e^{(-\ln 2 exp[(R_s - b_c)/a])}$,

$$R_s \approx 1.4(A_p^{1/3} + A_t^{1/3})fm$$

'strong absorption radius'.

Transfer to the continuum

First order time dependent perturbation theory amplitude:

$$A_{fi} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt < \phi_f(\mathbf{r})|V(\mathbf{r})|\phi_i(\mathbf{r}-\mathbf{R}(t)) > e^{-i(\omega t - mvz/\hbar)} \qquad (1)$$

$$\omega = \varepsilon_i - \varepsilon_f + \frac{1}{2}mv^2$$

$$\frac{dP_{-n}(b_c)}{d\varepsilon_f} = \frac{1}{8\pi^3}\frac{m}{\hbar^2 k_f}\frac{1}{2l_i+1}\Sigma_{m_i}|A_{fi}|^2$$
$$\approx \frac{4\pi}{2k_f^2}\Sigma_{j_f}(2j_f+1)(|1-\bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2)\mathcal{F},$$

$$\mathcal{F} = (1 + F_{l_f,l_i,j_f,j_i})B_{l_f,l_i} \qquad B_{l_f,l_i} = \frac{1}{4\pi}\left[\frac{k_f}{mv^2}\right]|C_i|^2\frac{e^{-2\eta b_c}}{2\eta b_c}M_{l_f l_i}$$

Wave functions

Final continuum state:

$$\phi_f(\mathbf{r}) = C_f k \frac{i}{2}(h_{l_f}^{(+)}(kr) - \bar{S}_{l_f} h_{l_f}^{(-)}(kr)) Y_{l_f,m_f}(\Omega_f),$$

$\bar{S}_{l_f}(\varepsilon_f)$ is an optical model n-t (n-core in fragmentation reactions) S-matrix.

or using the potential $V = V_{\rm nt} + V_{\rm eff}$ sum of the neutron-target optical and Coulomb potentials, a distorted wave of the eikonal-type

$$\phi_f(\mathbf{r},\mathbf{k}) = exp\left\{i\mathbf{k}\cdot\mathbf{r} + i\chi_{\rm eik}(\mathbf{r},t)\right\} \tag{2}$$

the eikonal phase shift is simply

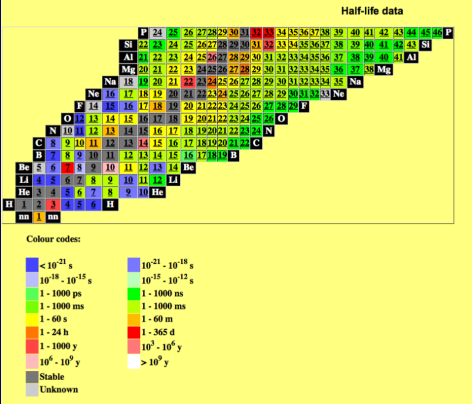
$$\chi_{\rm eik}(\mathbf{r},t) = \frac{1}{\hbar}\int_t^\infty V(\mathbf{r},\mathbf{R}(t'))dt'. \tag{3}$$

Initial state:

$$\phi_i(\mathbf{r}) = -C_i i^l \gamma h_{l_i}^{(1)}(i\gamma r) Y_{l_i,m_i}(\Omega_i).$$

L. Lo Monaco and DM Brink JPG11, 935, 1985; A. Mukhamedzhanov PRC 84, 044616, 2011; I. Thompson talk at DREB2012 (Pisa).

Entering the world of exotic nuclei



- Is there a life behind the dripline?
- Extend our understanding of the nuclear force.
- Check the limits of validity of structure models.
- In practice AFA my talk is concerned: try do do spectroscopy in extreme conditions.

Early experiments

I. Tanihata et al., Phys. Lett. B 160, 380 (1985)

$$\sigma_I = \pi\,[R_I(\mathrm{P}) + R_I(\mathrm{T})]^2$$

$$\sigma_R = \pi\left(R_{vol} + R_{surf}\right)^2\left(1 - \frac{B_c}{E_{cm}}\right)$$

Kox et al. (1987)

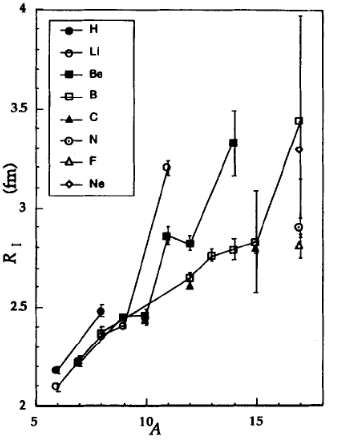
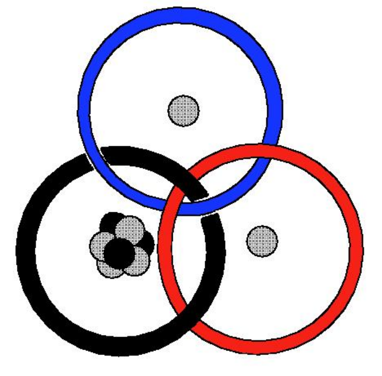


Fig. 2.1 Interaction cross sections of light nuclei determined by 800A MeV reactions.

Early theories

P. G. Hansen and B. Jonson, Europhys. Lett., 4, 409 (1987).



- Large spatial extension
- One- or two- neutron halo nuclei
- Two neutron halo (Borromean Systems): ^{6}He, ^{11}Li, ^{14}Be
- Three-body model
- Importance of the n-core interaction n-^{9}Li, n-^{12}Be $(+)$

Reaction theory: K. Yabana, Y. Ogawa,Y. Suzuki (1992); H. Sagawa, N.Takigawa (1994), H. Esbensen and GF. Bertsch; C. Bertulani and W.Bauer; M. Hussein and KW. MacVoy; G. Hansen(1996). Structure theory: Zhukov and Thompson, Surrey group, Nordic collaboration

Early eikonal model

$$\sigma_R = \int_0^\infty d\mathbf{b}(1 - |S(\mathbf{b})|^2) = \sigma_{ct} + \sigma_{nt}$$

decoupling of core and halo

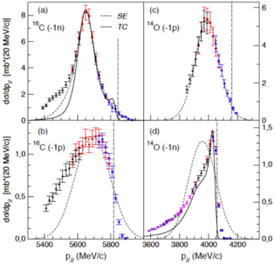
$$\begin{aligned} |S(\mathbf{b})|^2 &= e^{-[\sigma_{nn}\int ds\rho_p(|b-s|)\rho_t(s)]} \\ &= e^{-[\sigma_{nn}\int ds\rho_c\rho_t]}e^{-[\sigma_{nn}\int ds\rho_n\rho_t]} \\ &= |S_{ct}|^2|S_{nt}|^2 \\ &= e^{-[\sigma_{nn}\int ds\rho_c\rho_t]}(1 - \sigma_{nn}\int ds\rho_n\rho_t) \\ &= |S_{ct}|^2 - |S_{ct}|^2 P_{b_{up}} \end{aligned}$$

$$\sigma_{nt} = \int_0^\infty d\mathbf{b}|S_{ct}|^2 P_{b_{up}}$$

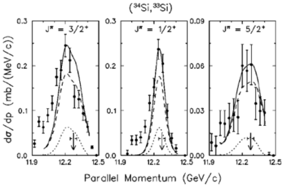
(!)⊗

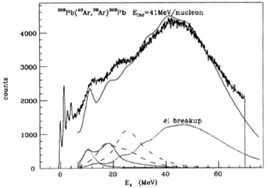
Knockout

F. Flavigny, A. Obertelli, AB et al., PRL 108, 252501 (2012). $**$



J.Enders et al. PHYSICAL REVIEW C **65** 034318





AB et al., PRC49(1994) 329

(+)

Kinematics

From Eq.1 $\otimes$ by the change of variables $dtdxdydz \rightarrow dxdydzdz'$ $e^{-i(\omega t - mvz/\hbar)} \rightarrow e^{-ik_1 z'} e^{ik_2 z}$ neutron energies to neutron parallel momenta with respect to core

$$k_1 = \frac{\varepsilon_f - \varepsilon_i - \frac{1}{2}mv^2}{\hbar v};$$

to target

$$k_2 = \frac{\varepsilon_f - \varepsilon_i + \frac{1}{2}mv^2}{\hbar v};$$

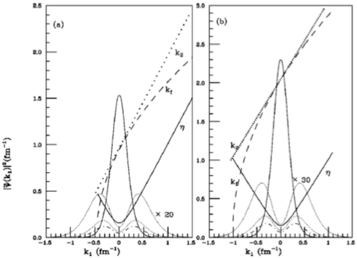
to core parallel momentum

$$P_{//} = \sqrt{(T_p + \varepsilon_i - \varepsilon_f)^2 + 2M_r(T_p + \varepsilon_i - \varepsilon_f)}, \quad (4)$$

breakup threshold at $\varepsilon_f = 0$

**

Example of kinematical effects PRC60(1999) 054604,PRC44(1991) 1559



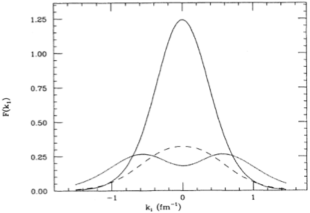


FIG. 11. Initial-state momentum distributions in ^{20}Ne according to Eq. (2.3a). The solid curve is for the $2s_{1/2}$ state, the dashed curve is the for $1p_{1/2}$, while the dotted curve is for the $1d_{5/2}$ state.

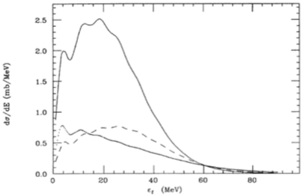


FIG. 7. Calculated total spectrum of the reaction ^{208}Pb(^{20}Ne,^{19}Ne)^{209}Pb at $E_{inc}=40$ MeV/nucleon. The solid curve is for the $2s_{1/2}$ initial state, the dashed curve is for the $1p_{1/2}$ initial state, while the dotted curve is for the $1d_{5/2}$ initial state.

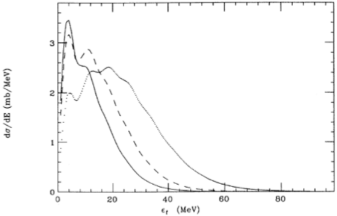


FIG. 8. Calculated total spectrum of the reaction ^{208}Pb(^{20}Ne,^{19}Ne)^{209}Pb for the $2s_{1/2}$ initial state. The solid curve is at $E_{inc}=25$ MeV/nucleon, the dashed curve is at $E_{inc}=30$ MeV/nucleon, and the dotted curve is at $E_{inc}=40$ MeV/nucleon.

(+)

Example of kinematical effects

AB and GF Bertsch, PRC63(2001) 044604; F. Flavigny et al., PRL 108, 252501 (2012).

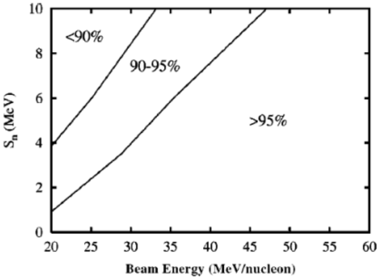


FIG. 1. Ratio of phase space integrals with and without momentum cutoffs, for a *d*-wave neutron wave function. The effect of the cutoff is to include less than 90%, between 90% and 95%, and more than 95% of the initial momentum distribution as marked on the figure.

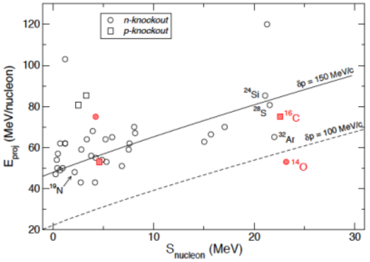


FIG. 3: (Color online) Nucleon-removal experiments from the literature [7, 22, 34] plotted as a function of the energy per nucleon of the projectile and the separation energy of the removed nucleon. The lines correspond to cutoffs appearing at $\delta p = 100$ and 150 MeV/c with respect to the center of the SE distribution. Data from the present experiment are in red.

Eikonal limit

Small neutron scattering angles

$$M_{l_f l_i} \approx P_{l_i}(X_i) P_{l_f}(X_f); \qquad P_{l_f}(X_f) \to I_0(2\eta \mathbf{b_v})$$

large n-t angular momenta

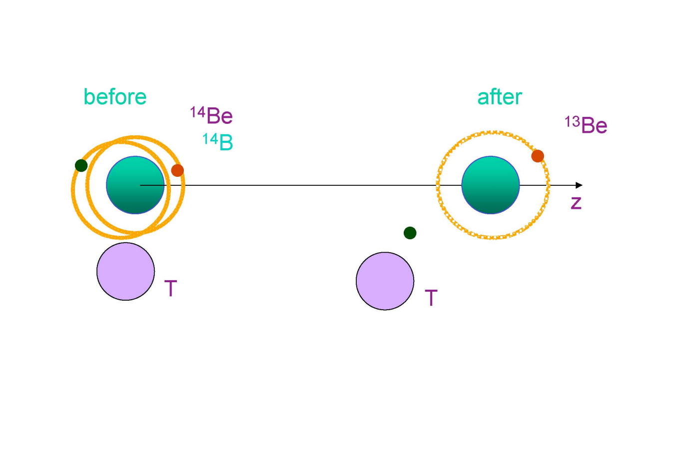
$$\frac{4\pi}{2k_f^2}\Sigma_{j_f}(2j_f+1) \to \int_0^\infty d\mathbf{b_v}$$

both conditions might not be well satisfied for stripping of deeply bound nucleons unless the core-target scattering is very peripheral. If possible verify core angular distributions.

$$P_{-n}(\mathbf{b_c}) = \int_0^\infty d\mathbf{b_v}(|1-\bar{S}(b_v)|^2 + 1 - |\bar{S}(b_v)|^2)|\tilde{\phi}_i(|\mathbf{b_v}-\mathbf{b_c}|, k_1)|^2$$

Notice $k_1 \to -\infty$ not strictly necessary.

Fragmentation reaction (coincidence)



$*\,*\,*\quad(+)$

Fragmentation of a 2n-halo nucleus $***$

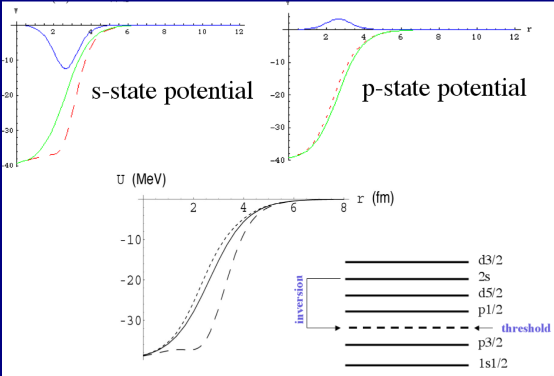
Inelastic-like excitations can be described by the first order time dependent perturbation theory amplitude $\otimes$:

$$A_{fi} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt \langle \psi_f(t)|V_2(\mathbf{r}-R(t))|\psi_i(t)\rangle$$

This method has the advantage that different potentials can be used for the determination of ψ_f and ψ_f.

Which components of the initial wave function show up in the continuum??

Determination of the bound and unbound (via optical model n-core S-matrix) states.



$$U(r) = V_{WS} + \delta V$$

$$\delta V(r) = 16\alpha \frac{e^{2(r-R)/a}}{(1+e^{(r-R)/a})^4}$$

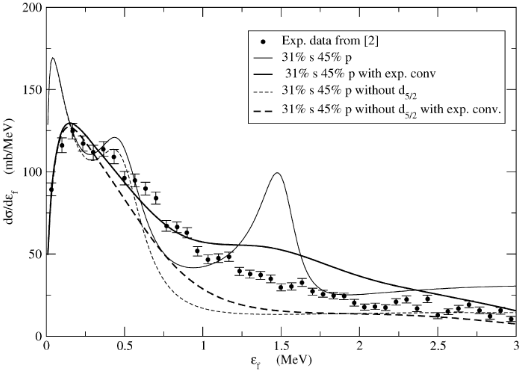
V_{WS} = Woods-Saxon + Spin orbit
δV = Correction to the potential originated from p.v. coupling (*N. Vinh Mau and J. C. Pacheco, NPA607 (1996) 163*)

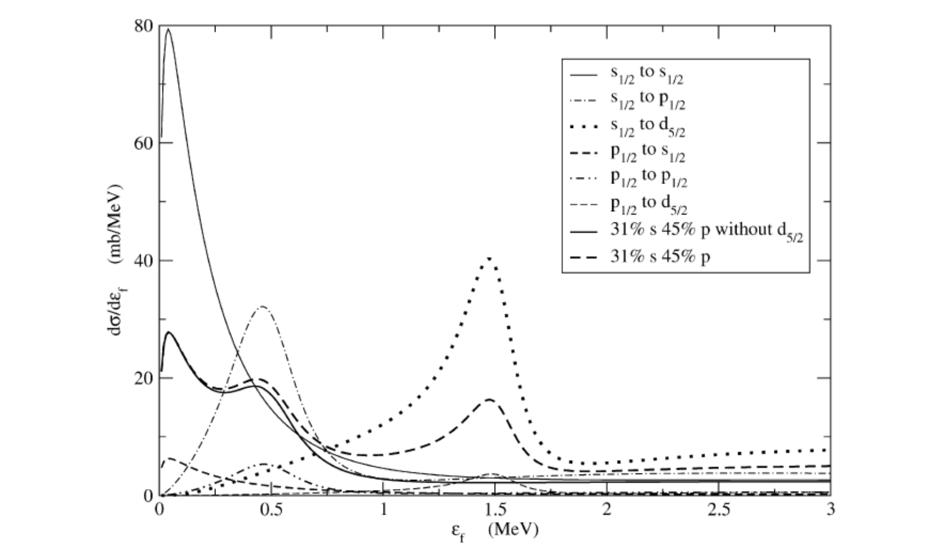
Fragmentation (^{10}Li best example)

GSI, H. Simon et al. NPA791 (2007) 267; G. Blanchon et al. NPA791 (2007) 303

Table 3
Scattering length of the 2s continuum state, energies and widths of the *p*- and *d*-resonances in ^{10}Li and corresponding strength parameters for the δV potential

	ε_{res} (MeV)	Γ_j (MeV)	a_s (fm^{-1})	α (MeV)
$2s_{1/2}$			−17.2	−10.0
$1p_{1/2}$	0.63	0.35		3.3
$1d_{5/2}$	1.55	0.18		−9.8





80
60
40
20
0
0
0.5
1
1.5
2
2.5
3
$d\sigma/d\varepsilon_f$ (mb/MeV)
ε_f (MeV)
$s_{1/2}$ to $s_{1/2}$
$s_{1/2}$ to $p_{1/2}$
$s_{1/2}$ to $d_{5/2}$
$p_{1/2}$ to $s_{1/2}$
$p_{1/2}$ to $p_{1/2}$
$p_{1/2}$ to $d_{5/2}$
31% s 45% p without $d_{5/2}$
31% s 45% p

^{14}Be structure

The ground state of ^{14}Be has spin $J^{\pi} = 0^{+}$. In a simple model assuming two neutrons added to a ^{12}Be core in its ground state the wave function is:

$$|^{14}Be >= [b_1(2s_{1/2})^2 + b_2(1p_{1/2})^2 + b_3(1d_{5/2})^2] \otimes |^{12}Be, 0^+ >$$

Then the bound neutron can be in a 2s, $1p_{1/2}$ or $1d_{5/2}$ state. However, the situation is much more complicated and in particular the calculations of Tarutina, Thompson and Tostevin show that there is a large component $(2s_{1/2}, 1d_{5/2}) \otimes |^{12}Be, 2^+ >$ with the core in its low energy 2^+ state which can modify the neutron distribution.

^{13}Be experimentally

It is experimentally proved that

- ^{13}Be is not bound
- $5/2^+$ resonance at 2MeV
- $S_{2n}(^{14}\text{Be}) = 1.34 \pm 0.11$MeV

Fragmentation: (^{13}Be puzzle)

(a)

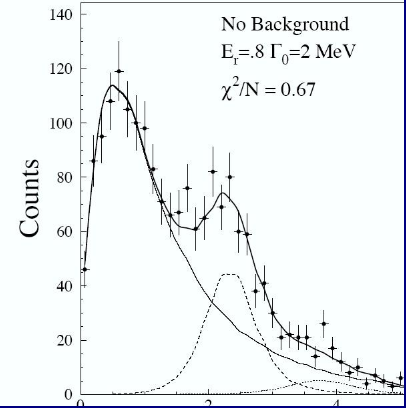


Figure: (a) LPC & GANIL, Lecouey, Orr et al. 2002.

(b)

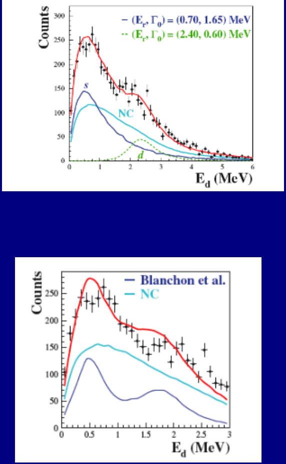


Figure: (b) LPC & GANIL, G. Randisi, N.Orr et al. 2012, DREB12's talk and private comm.

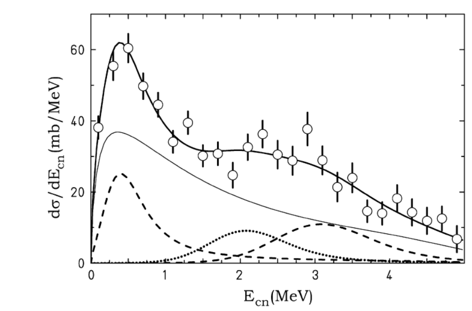


Figure: (a) GSI, H. Simon et al. NPA791 (2007) 267.

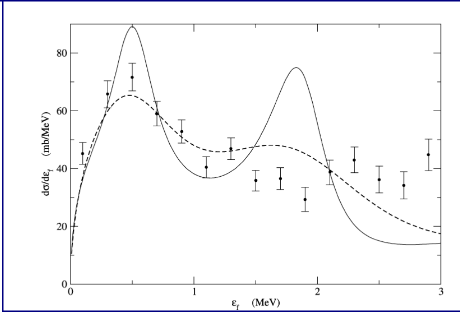


Figure: (b) G.Blanchon et al. NPA784 (2007) 49.

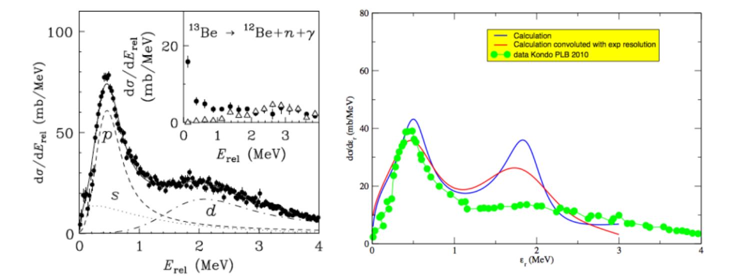


Figure: (b) RIKEN, Y. Kondo et al. PLB690 (2010) 245; G. Blanchon, private communication.

^{12}Be and ^{14}Be in pp-RPA: inversion (B), non inversion (A)

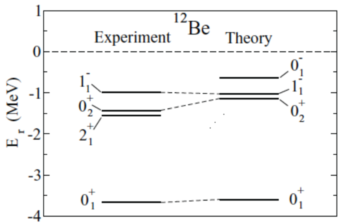


FIG. 1. Low-lying spectra of ^{12}Be obtained with pp-RPA compared to experiment.

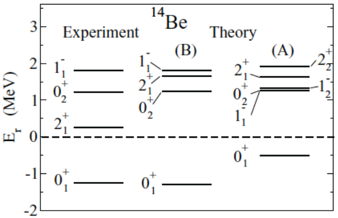
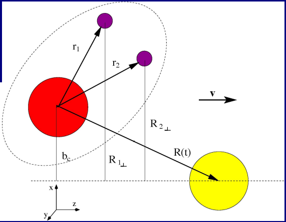


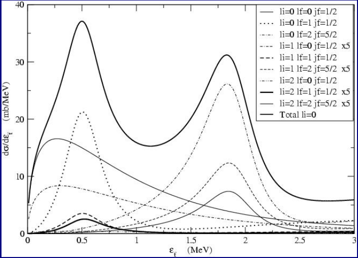
FIG. 3. Low-lying spectra of ^{14}Be obtained with pp-RPA without (A) and with (B) inversion in ^{13}Be compared to experiment.

	$S_{2n}(^{12}\mathrm{Be})$	$S_{2n}(^{14}\mathrm{Be})$	$\langle \mathbf{r}^2\rangle^{1/2}_{A+2}$	$\langle \rho^2\rangle^{1/2}$	$\langle \lambda^2\rangle^{1/2}$
A	2.91	0.51	3.45	8.45	5.45
B	3.71	1.29	2.91	4.56	4.02
Experimental	3.673 ± 0.015	1.26 ± 0.01	3.10 ± 0.15	5.4 ± 1.0	4.2 ± 1.7

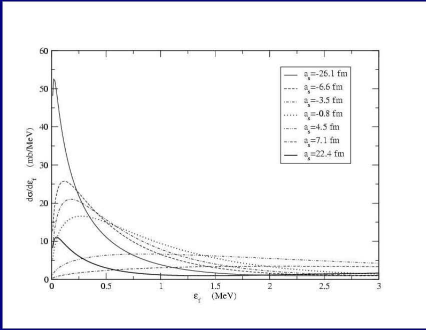


Strength of every transition

Gives information about the 'mother' nuclei



Dependence on the scattering length of the final s-state



Conclusions

- Time dependent methods have been successful for about twenty years in describing breakup in very different situations. Kinematics is important, as expected. The eikonal can be obtained as a limiting case.
- There are very good indications that in the ^{13}Be spectrum the shell ordering is 2s, $p_{1/2}$ and $d_{5/2}$ with a shell inversion.
- The shell model seems to be working well as high as 40MeV in the continuum as down to 20MeV bound states in exotic nuclei.

Past-Present-Future

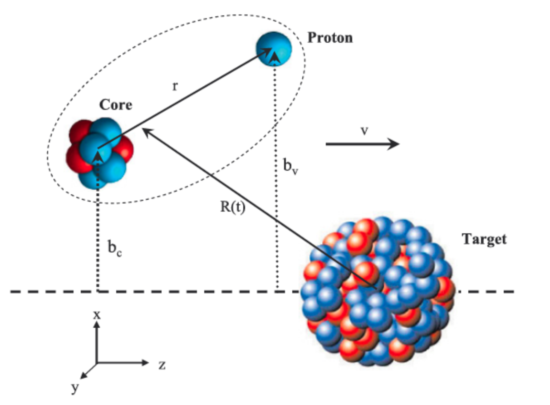
Possible experimental developments:

- Knockout: kinematical complete experiments with reconstruction of target final state.
- Projectile-fragmentation...perhaps the most difficult experiments to make...and interpret?
- The past has been characterized by studies at high incident energy and for weakly bound projectiles. In the future more and more strongly bound nuclei will be studied at lower energies at ISOL-type facilities.

Some of my co-authors, besides George:

D. M. Brink
N. Vinh Mau
G. Blanchon
F. Flavigny, A. Obertelli

Sketch of coordinates



Different shapes and widths

E. Sauvan, F. Carstoiu et al., PRC69, 044603 (2004)

E. SAUVAN *et al.* PHYSICAL REVIEW C 69, 044603 (2004)

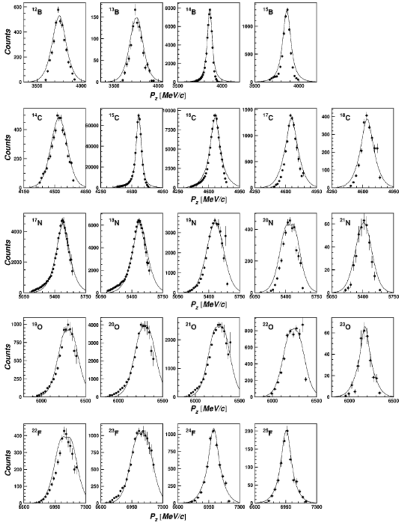
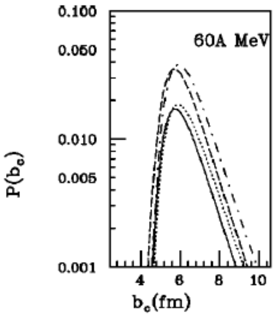


FIG. 1. Comparison of the core fragment longitudinal momentum (p_z) distributions obtained using a carbon target and the Glauber model calculations (solid line).

Hankel-effect

AB and GF Bertsch, PRC63(2001) 044604



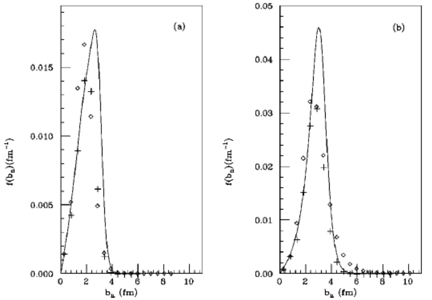


FIG. 5. The integrand function of the diffraction (a), and stripping (b) term of Eq. (11) after k_z integration, full curve, obtained from the realistic bound state wave function and the corresponding terms, diamonds, in the sum over partial waves of Eqs. (5) and (6) in the case of the incident energy of 78*A* MeV. Crosses are the results of a calculation of Eq. (11) in which the eikonal phase shifts have been substituted by the optical model phase shifts as in Eq. (B9). All calculations done at fixed impact parameter b_c=5.6 fm between the projectile and target.